ANALOG MODEL FOR PILES IN EXPANSIVE CLAY

by

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Abstract

This study relates to the use of the double analogy between the swelling of clays caused by moisture movement, and the thermal expansion of solids which accompanies heat transfer. An analog thermoelastic model was constructed and operated to study how piles in expansive clays are effected by seasonal moisture changes.

In spite of its limitations, the model demonstrated pile behaviour characteristics, some of which were previously found in laboratory models and field observations.

1. Introduction

The combination of expansive clays and a semi-arid climate, where wet and dry seasons alternate, causes considerable movement of the ground, with frequent distress to structures based on it. Modern building codes recognize this phenomenon and specify piled foundations, and sometimes also peripheral impervious membranes, as a possible remedy. Yet, the continuing occurence of foundation failures in expansive clays only proves that knowledge in this respect is still imperfect.

The behaviour of piles in expansive clays was studied through small-scale laboratory models (3,5). In these models, the piles were surrounded by compacted clay, and the effect of full saturation on their behaviour was measured. The results, therefore, are only of limited applicability. The aim of this paper is to present a different kind of model, where the performance of piles subject to correct environmental boundary conditions may be readily studied.

2. Principles of the Analog Model

2.1. The Double Analogy

It was shown (2) that seasonal moisture movement in expansive clay approximately obeys the linear diffusion equation:

$$\frac{\partial w}{\partial t} = D \nabla^2 w \qquad (1)$$

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where w is the moisture content, t the time coordinate and D the moisture diffusivity. When w is substituted by temperature T, equation (1) becomes the well-known heat equation, which governs heat transfer in solids. Further, it is known (2) that volume changes of expansive clays are linearly-dependent on their moisture content, and thus are analogous to the thermal expansion of solids. This double analogy served as a basis for the analog thermo-elastic model described in this paper. In this model, cyclical heat flux, applied to the upper surface, simulated the annual cycle of wetting and drying. The resulting temperature distribution, as well as displacements in the components representing the expansive clay and the concrete pile, were measured for different types of piles, enabling a comparison and evaluation of their performance.

2.2 Description of the System

The model (Fig. 1) was designed to study the behaviour of a pile supporting the outside wall of a typical residential building. For practical reasons, only a slice of ground, perpendicular to the wall and containing the pile, was considered. The model was constructed in the form of a lead plate, 30 mm. thick, 1000 mm. long and 800 mm. wide, standing on its long edge. The lead, which is relativley soft and with a high coefficient of thermal expansion, simulated the expansive clay. Having a scale of 1:15, this represented a region 12 m. deep, and with a horizontal dimension of 15 m. Forty-eight thermo-couples were installed in characteristic points. A vertical steel bar, with a section of 30 by 32 mm., was embedded in the lead at a distance of 265 mm. from the edge and represented a concrete pile. The vertical edges of the plate were fixed against horizontal movement by a rigid steel frame. This frame also served as a base for LVDT (Linear variable differential transducers), which measured displacements of the pile and the top edge of the plate, with accuracy better than 1μ. The output of the LVDT was filtered, amplified and recorded.

The exposed part of the upper edge was heated by radiation from two infra-red tubes, mounted in parabolical reflectors. Variable power was supplied to the tubes by a special electro- mechanical device , incorporating an auto-transformer and having a cycle period of 102.5 sec. Except for the heated edge, all other surfaces were insulated by rock wool, with a thickness of 300 mm. The total system is shown in Fig. 2.

2.3 Thermal Similitude

If w_o, t_o, and L_o are characteristic moisture, time and length values of the system, respectively, equation (1) may be normalized as follows:

$$\frac{\partial \omega}{\partial \tau} = D \frac{t_o}{(Lo)^2} \nabla^2 \omega \qquad (2)$$

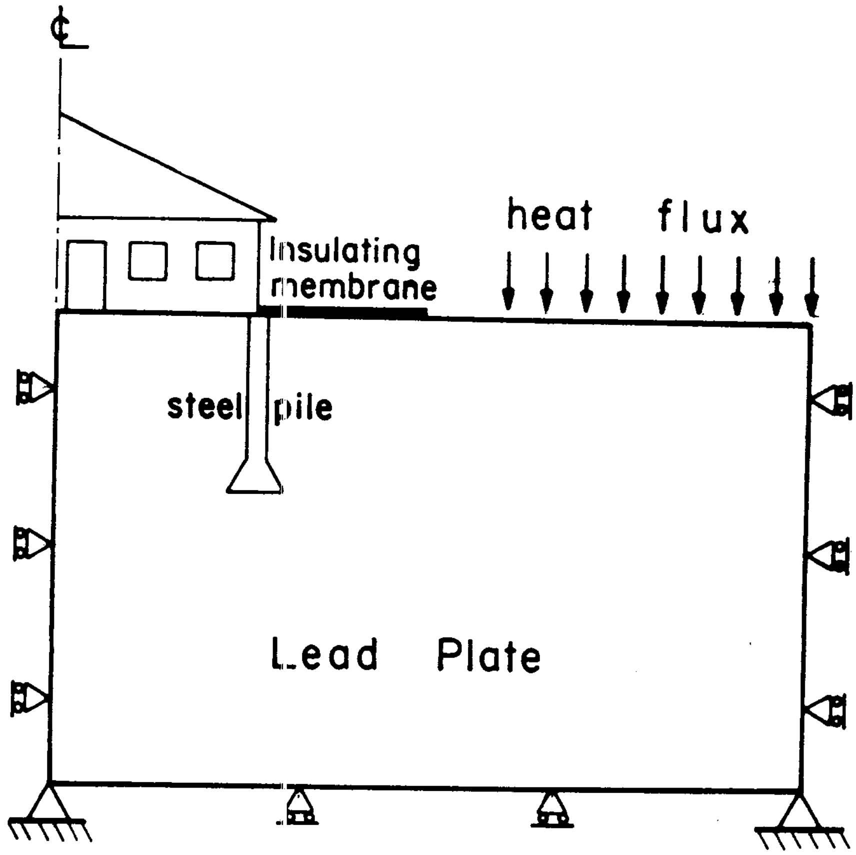


Figure 1: Scheme of the Model

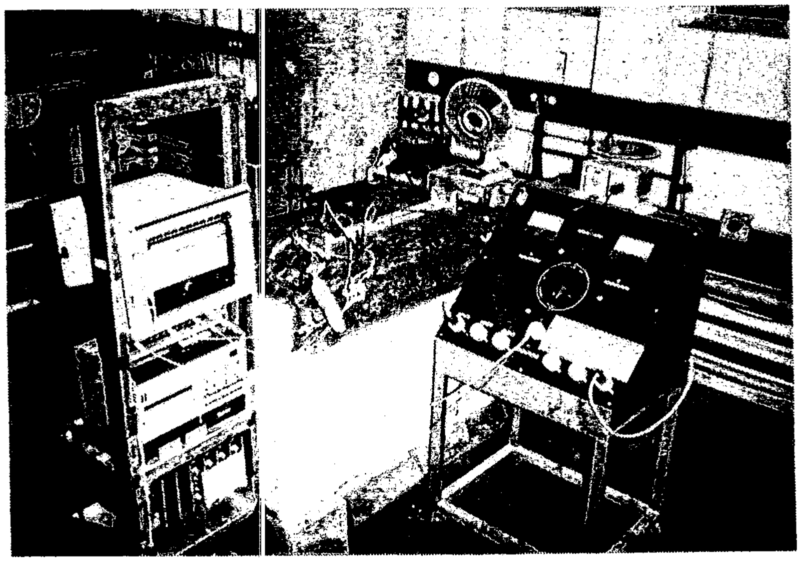


FIG. 2: GENERAL VIEW OF THE SYSTEM

where $\omega = w/w_o$ and $\tau = t/t_o$. If ω is replaced by normalized temperature θ, and D by thermal diffusivity δ, one gets the normalized diffusion equation:

$$\frac{\partial\theta}{\partial\tau} = \delta \frac{t_o}{(Lo)^2} \nabla^2\theta \qquad (3)$$

For similitude to exist between heat conduction in the model m and moisture movement in the prototype p, the following conditions must be satisfied :

1. Geometrical similitude (L_{om}/L_{op} = constant)
2. Boundary conditions similitude ($\Theta_b = \Theta_b$)
3. Equal Fourier Numbers ($\delta t_{om}/(L_{om})^2 = Dt_{op}/(L_{op})^2$)

If such is the case, the identity $\Theta = \omega$ exists for homologous points at homologous times.

From condition 3, the time scale of the model is as follows:

$$\frac{t_{op}}{t_{om}} = \frac{\delta}{D} \left(\frac{L_{op}}{L_{om}} \right)^2 \qquad (4)$$

The diffusivities of lead and clay are 0.238cm^2/sec and 1.6 x 10^{-4} cm^2/sec, respectively (9). Substituting in equation (4) gives:

$$\frac{t_{op}}{t_{om}} = \frac{0.238}{1.6 \times 10^{-4}} \times 15^2 = 334.700 \qquad (5)$$

2.4. Kinematic Similitude

The thermoelastic equations for plane stress are (7):

$$\varepsilon_{11} = \frac{1}{E}(\sigma_{11} - \nu\sigma_{22}) + \alpha T$$
$$\varepsilon_{22} = \frac{1}{E}(\sigma_{22} - \nu\sigma_{11}) + \alpha T \qquad (6)$$
$$\varepsilon_{12} = \frac{1+\nu}{E}\sigma_{12}$$

Where E is Young's Modulus, ε denotes the strains σ - the stresses, ν - Poisson's Ratio and α - the coefficient of thermal expansion. With no external forces, stresses in the region are caused only by temperature distribution, and because of linearity:

$$\sigma_{11} = A\,\alpha\,ET$$
$$\sigma_{22} = B\,\alpha\,ET \qquad (7)$$
$$\sigma_{12} = C\,\alpha\,ET$$

Where the dimensionless coefficients A,B and C depend on the boundary conditions and temperature distribution. Substituting in (6) and rearranging one gets:

$$\varepsilon_{11} = \alpha T_o \ [\Theta(A - \nu B + 1)]$$
$$\varepsilon_{22} = \alpha T_o \ [\Theta(B - \nu A + 1)] \qquad (8)$$
$$\varepsilon_{12} = \alpha T_o \ [\Theta(C + \nu\sigma)]$$

For kinematic similitude to exist, the expressions in the square brackets must be equal in the model and the prototype, therefore Poisson's Ratios must be identical. In this case, strains are related as $\alpha \ T_o$, and displacements as $\alpha \ T_o L_o$.

Because the prototype is essentially plane-strain, while the model is plane stress, the following modification of Poisson's ratio must be made (7):

$$\nu_m = \frac{\nu_p}{1-\nu_p} \qquad (9)$$

For $\nu_m = 0.42$, $\nu_p = 0.3$, which is typical for the expansive clay. Therefore, thermal similitude ensures kinematic similitude.

2.5 Application of Cylical Heat-Flux

Seasonal moisture flux on the clay surface is approximately a sinusoidal function of time (1). When sinusoidal heat-flux is applied to the upper boundary of the model, temperature distribution inside the plate is (4):

$$T(z,t) = \bar{T} + T_m e^{-\sqrt{\omega/2\delta}z} \sin(\omega t - \sqrt{\omega/2\delta}z) \qquad (10)$$

where ω is the circular frequency, $\bar{T}$ the average temperature on the boundary, T_m the temerature amplitude, and z the depth. Denoting $\sqrt{\omega/2p} = p$ and differentiating, the gradient at the upper surface is:

$$\frac{\partial T}{\partial z} (0,t) = - p \ T_m(\sin \omega t + \cos \omega t) \qquad (11)$$

From the total flux q emmitted by the heating element, some is wasted to the atmosphere, and the rest, denoted by qM, hits the upper edge of the plate. Out of this, one part penetrates the model:

$$q_{cond.} = - \kappa \frac{\partial T}{\partial z} = \kappa p \ T_m(\sin \omega t + \cos \omega t) \qquad (12)$$

Where κ is the thermal conductivity. The rest of the heat is radiated to the atmosphere, the temperature of which is T_∞:

$$q_{conv} = h\ (T - T_\infty) + h\ T_m \sin \omega t \qquad (13)$$

where h is the heat-transfer coefficient. Adding together:

$$qM = q_{cond.} + q_{conv.} = h(T-T_\infty) + (h + \kappa p)T_m \sin \omega t + \kappa p\ T_m \cos \omega t \qquad (14)$$

If the heated area is A , and the resistance of the heating element R , the voltage applied to this element must be:

$$V = \sqrt{q\ A\ R} \qquad (15)$$

For moving air, h is between 2 and 50 Btu/hr ft^2 ^{o}F (4). Substituting this, as well as the correct values for ω and p, one gets the maximum voltage when the phase angle ωt is between 45.2^o and 50.8^o.

From equation (10), the maximum temperature on the edge appears when $\omega t = 90^o$. Therefore, the temperature should lag 39.2^o to 44.8^o behind the voltage function, or between 11.2 and 12.7 seconds.

3. Test Program

Altogether, six tests were carried out (table no.1):

Table no. 1: Test Details

Test no.	pile length(mm)	enlarged base	external membrane	remarks
101	265	no	no	-
102	265	no	135 mm	-
103	265	no	265 mm	-
104	500	no	no	-
105	500	no	no	pile exposed on both sides
106	265	yes	0+135mm+265mm	membranes introduced under reflectors during test

In all tests, pile heads were not restrained, and the unheated part of the upper edge was insulated with 50 mm. of rock wool. In test no. 106 the membrane thickness was 10 mm., with aluminum paper on top.

Before each test, the plate was heated to the desired uniform temperature (about 200°C) using, in addition to the infra-red heater on top, also a thermostatically-controlled heating element under the bottom of the plate. After 24 hours, when temperatures at both edges were found to be equal, the frame was clamped, and the variable power supply started.

4. Results-Plate Behaviour

4.1. Temperature Field Uniformity

Ideally, the average temperature should be uniform all over the plate. In practice, however, a temperature spread of about 80°C was detected (Fig. 3). This lack of uniformity, due to unavoidable imperfections in the insulation, is typical for a plate with heat sources in the upper and lower edges, and sinks in the vertical edges.

Considering this temperature gap and the linearity of the thermocouples, all measurements were made relative to room temperaure.

4.2. Temperature Changes

4.2.1. Magnitude

In all tests (Fig. 4) the temperature of the heated surface varied as a sinusoidal function of time. For an average temperature of 185°C, peak-to-peak amplitudes of 10°C to 24°C were measured.

According to theory (9), temperature amplitude under the edge of a membrane is half the value found on an exposed surface. Such a result was found in tests No. 102 and 106, while in test No. 103 the respective ratio was 1:3. A possible explanation for this inconsistency is that the location of the thermocouple did not coincide exactly with the edge of the membrane.

4.2.2 Timing

On the average, temperature on the surface showed a phase lag of 25.3 sec. behind voltage, compared to the theoretical Value of 11.2 to 12.7 sec. The difference may be attributed partly to the heat capacity of infra-red tubes, and partly to the slowness of the thermocouple recorder.

The curves of surface temperature served to define seasons, as it is known (1) that maximum surface moisture (temperature in the model), is reached after three quarters of the winter has elapsed.

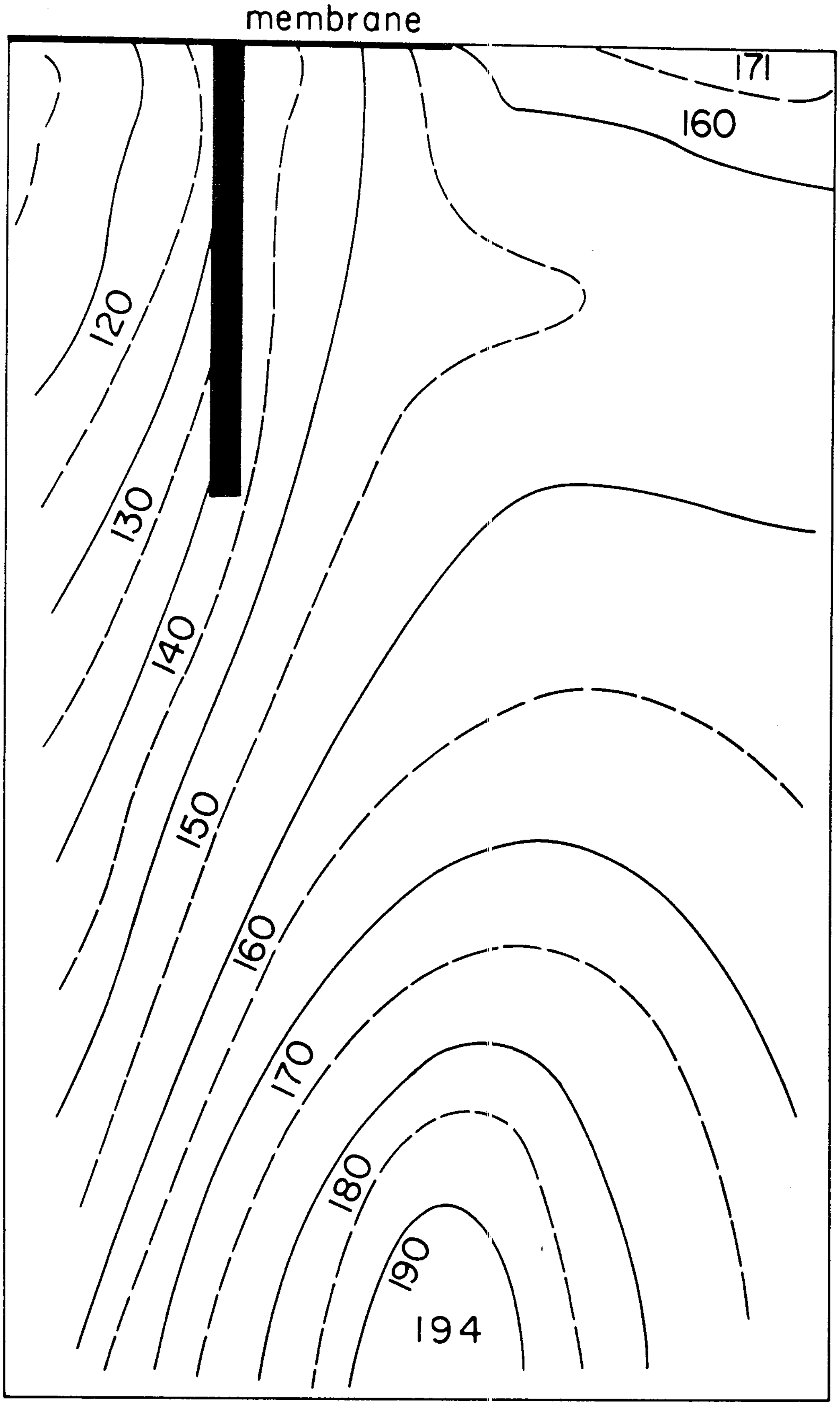
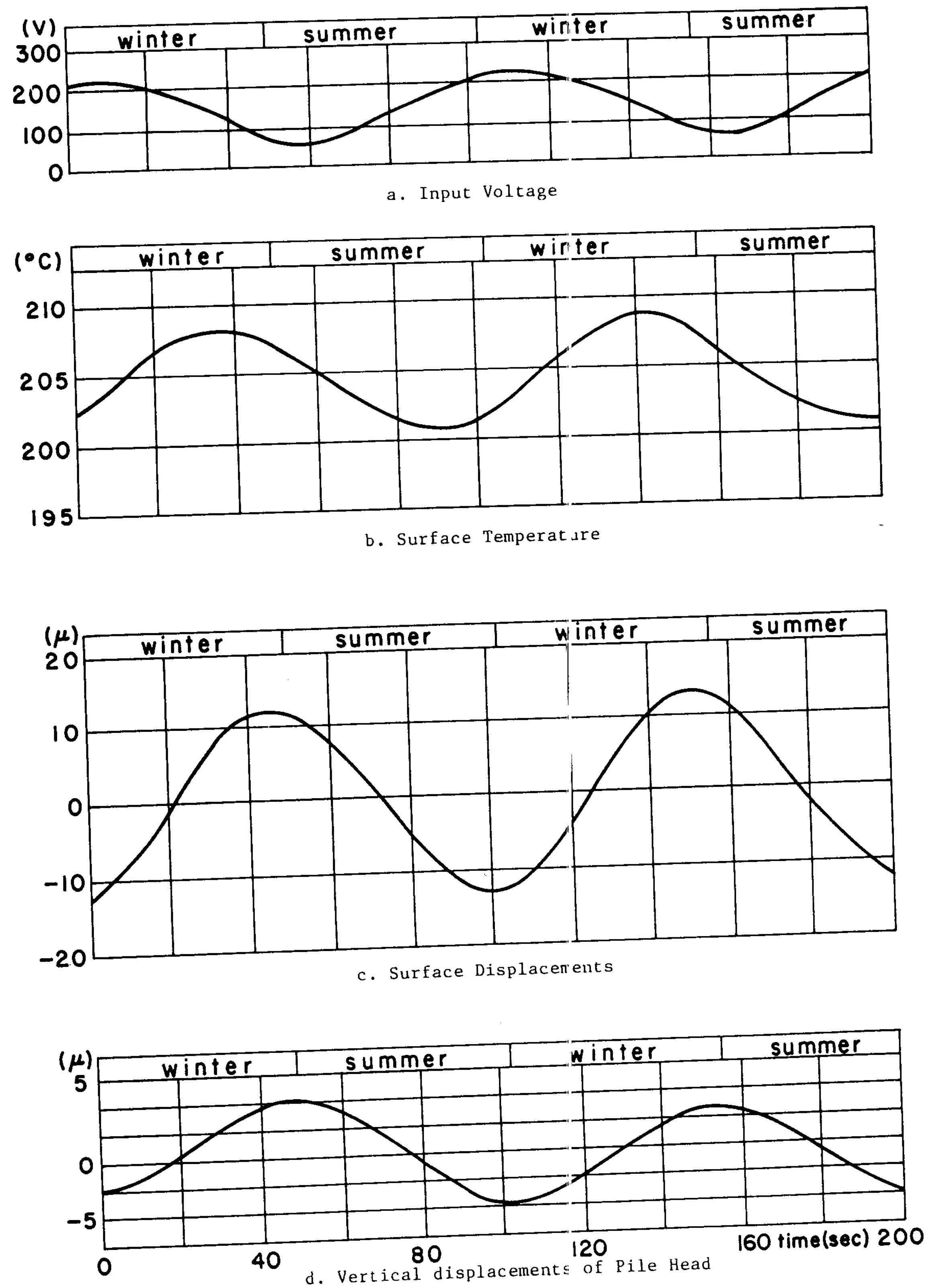


Fig. 3: TEMPERATURE DISTRIBUTION IN THE PLATE
(DEGREES CENTRIGRADE - DISTORTED SCALE)



a. Input Voltage

b. Surface Temperature

c. Surface Displacements

d. Vertical displacements of Pile Head

FIG. 4: PILE No. 105 -- TEST RESULTS

4.3. Edge Displacements

4.3.1. Magnitude

From theory (1), the surface displacement should be a sinusoidal function of time, with an amplitude:

$$S_m = 0.14\ L\alpha(1+\nu)T_m \tag{16}$$

where L is the depth of the active layer (2 m in the prototype). By substituting $T_m = 8^oC$, and the appropriate values of α and ν one gets:

$$S_m = 0.14\ \frac{2000}{15} \times 29.3 \times 10^{-6} \times (1 + 0.42) \times 5 = 6.0 \times 10^{-3}\text{mm} \tag{17}$$

The recorded displacement curves (Fig. 4) were, in fact, practically sinusoidal, but the measured amplitude was twice as large, proving that close to the upper surface, heat transfer is not two-dimensional as planned, but also takes place perpendicular to the plate, through the corners of the insulation.

4.3.2. Timing

From Fig. 4, it is seen that the surface displacement function lagged about 15 sec. behind the temperature, compared to the theoretical value of 12.8 sec. Considering the fact that the displacement signal was passes through a RC filter, the 2.2 sec difference seems reasonable.

5. Pile Behaviour

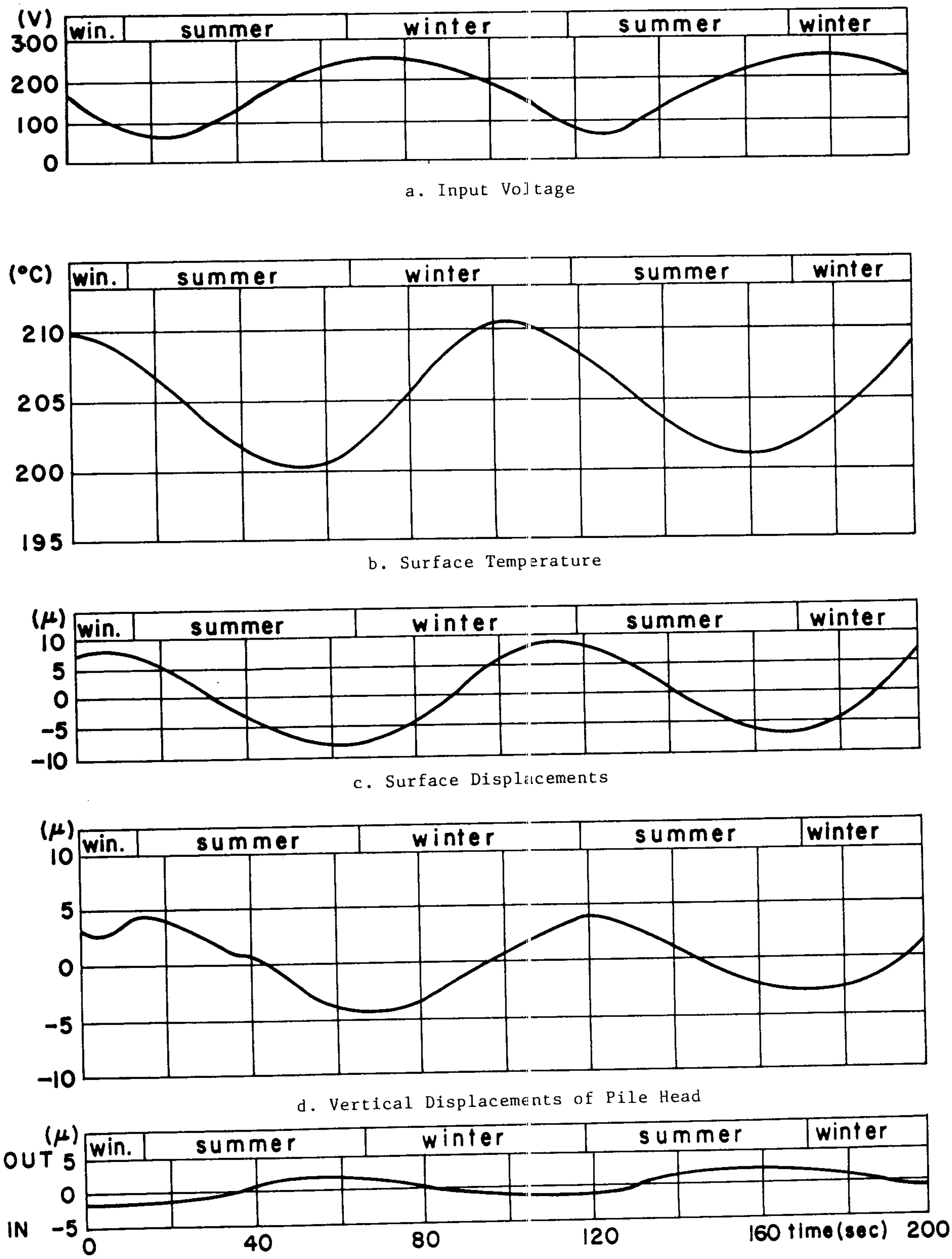
5.1. Single Pile

A pile was defined as single when the surface on both its sides was exposed to heat flux.

The results of test no.105 (Fig. 4) show that while the upper edge of the plate moved up and down with a peak-to-peak amplitude of 23μ , the respective movement of the pile head was only 9μ , or 39% . In addition, the displacements of the pile head lagged about 4 sec. (two weeks in an annual cycle) relative to the exposed surface. This reduction and lag of the pile movement in relation to the surface movement was observed both in the field (8) and in laboratory models (3).

5.2. Pile Under a Structure

The results of test no. 104 (Fig. 5) are typical for a pile which was subject to heat flux on one side only. Here too the pile showed a cyclical vertical movement, with an amplitude of 8μ (which is about one half of the surface movement), and a lag of 8 sec. In this case a cyclical horizontal motion with an amplitude of 7μ was also recorded. Generally, the pile head moved inside during winter and out-



a. Input Voltage

b. Surface Temperature

c. Surface Displacements

d. Vertical Displacements of Pile Head

e. Horizontal Displacements of Pile Head

FIG. 5: PILE NO. 104 - TEST RESULTS

side during summer. An interesting result is the difference in phase between both kinds of motion: The horizontal displacements lead by 14 sec. the vertical ones, and by 6 sec. the surface displacements.

To enable comparison between different pile types, the pile movements were normalized through division by the amplitude of the surface movement. In addition, the lag of the pile head movement behind that of the surface, divided by the cycle period, was defined as relative lag (lead is shown by a minus sign). The results of tests no. 101,104 and 106 are summarized in table no.2.

Table no. 2 - Summary of Test Results

Test no	pile length (mm)	under-ream	relative amplitude		relative lag	
			vertical	horizontal	vertical	horizontal
101	265	no	0.50	0.47	0.078	-0.059
104	500	no	0.46	0.19	0.049	0.059
106	265	yes	0.20	0.20	0.098	-

The above results show that lengthening the pile resulted mainly a reduction in the horizontal movement. The provision of an enlarged base, in comparison, reduced both horizontal and vertical displacements.

5.3 Influence of Impervious Membranes

Peripheral impervious membranes, which may serve to stabilize shallow footings (6), were found to increase the movement of deep foundations on expansive clay (2). Test no. 101 to 103 (straight shaft) and 106 (belled pile) were run to check the membrane effect in the analog model. The results here were contradictory:

In test no. 102 the membrane, with a width equal to one half the pile length, decreased the displacements of the pile head to one quarter of the values found without any membrane. A wider membrane (test no. 103) caused all displacements to practically disappear.

Test no. 106, on the other hand, demonstrated the negative effect of the membrane: A half-length membrane increased relative amplitude from 0.20 to 0.32,while the wider membrane (equal to one pile length), increased it further to 0.40. Relative amplitudes in the horizontal direction were 0.20, 0.16 and 0.19, respectively.

The results of test no. 106 seem more reliable, because in this pile the relative displacement was closest to real values, and because the membranes were changed while the model functioned, with all other parameters kept constant.

6. Conclusions

From the work described, certain conclusions may be drawn:
1. It is possible to construct and operate a thermo-elastic analog model to simulate the behaviour of piles in an expansive clay which is subject to seasonal moisture changes.
2. The model is able to demonstrate certain patterns of behaviour of piles, some of which were observed independently by other methods.
3. Due to unavoidable flaws in the insulation, and to slippage between "pile" and "soil", test results are often only quantitative.
4. The analog model is able to simulate elastic, homogeneous and isotropic clay, in which moisture movement is governed by the linear diffusion equation. With the availability of more exact soil parameters, the analog model will become inadequate, and different method, such as the finite-element method (2) will have to be used.

Acknowledgement

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Appendix 1 - References

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Appendix 2: List of Symbols

A - area, coefficient
B - coefficient
C - coefficient
D - moisture diffusivity
E - Young's modulus
h - heat-transfer coefficient
L - length, depth of active layer
q - heat flux
R - resistance
S_m - Amplitude of surface displacement
T - temperature
t - time
w - water content
z - depth
α - coefficient of thermal expansion
ν - Poisson's ratio
τ - normalized time
ω - normalized moisture content, circular frequency
Θ - normalized temperature
δ - thermal diffusivity
ε - strain
σ - stress
κ - thermal conductivity