A Lumped-Parameter Model for Statnamic Testing

by
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1. Introduction

The pile loading test is, in all likelihood, the most expensive geotechnical test in the book of ASTM standards (ASTM). Nevertheless, its results suffer from the same large scatter typical of most geotechnical tests. Moreover, even the results of a single test are usually open to different interpretations (Fellenius 1980). One may, therefore, safely conclude that pile load testing carries the worst cost/benefit ratio in the business.

The introduction of Statnamic testing improved this unfortunate situation by significantly lowering the cost of the test. It therefore seems now that the only way for additional gain is by achieving better understanding of the results.

Shortly after Statnamic testing made its debut, it became evident that the results obtained in this short-duration test do not necessarily represent the static behavior of piles. Since static loading is predominant in practice, it follows that the Statnamic results must be duly corrected for the time effect.

The existing methods for achieving this end are based on three distinct models:

1. The Equilibrium-point Method (Middendorp *et al.* 1992)
2. The Finite Element method (Yamashita *et al.* 1994)
3. An enhanced Smith Model (El Naggar & Novak 1992)

Since the validity of a given model can be demonstrated only by calibration against a corresponding static test, one faces an inherent problem:

1. If the comparison is made for the same pile, one of the tests must precede the other, rendering the calibration meaningless.
2. If, on the other hand, static and Statnamic tests are performed on different piles, comparison is of limited value unless statistical methods are employed. This means that a significant sample should be tested for each possible combination of pile- and soil-types. To the best of the authors' knowledge this has not yet been done and, with loading tests being such a costly matter, will probably remain undone.

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By this reasoning, none of the above methods may claim to have passed the necessary field trials. In addition, these methods are either complicated or theoretically questionable, or both (Amir 1995). There is thus a justification to present a new method, given that it is both simple and theoretically sound.

Unlike the El Naggar & Novak model, which simulates the pile-soil system by a large number of elements, the proposed method uses lumped-parameters (Richart *et al.* 1970) to represent the motion of the pile. Analyzing the Statnamic test results, the proposed method does not need any information regarding soil properties.

2. Basic Considerations

A typical load-displacement diagram of a *static* pile loading test usually takes the form of a convex curve, which can be approximated by the general form:

$$P = \frac{S^q}{a + b.S} \qquad (1)$$

In the particular case where q = 1 and b = 0, the spring is linear. If q = 1, the function is hyperbolic (Chin 1970) and If q = 0.5, the model is that proposed by Brinch Hansen (1963). Both extremes do not faithfully represent real load-displacement curves: in Chin's model, yield is reached only at infinite settlement. In Brinch Hansens' model, on the other hand, the tangent modulus is zero. It is thus a probability that the power of S lies in between 0.5 and 1.0.

When a pile is loaded dynamically, the load P(t) necessary to produce a given displacement S(t) depends not only on S(t), but also on both its first and second derivatives with respect to time (velocity and acceleration).

In a Newtonian liquid, stress is proportional to the rate of strain. There is no reason, however, to assume that a pile-soil system should demonstrate similar characteristics. In fact, there is evidence (Litkouhi & Poskitt 1980) that in such a system the resistance to the velocity factor (damping) is proportional to the 0.2 power of the velocity.

It is further assumed that the third factor (resistance to acceleration) is constant and equal to not less than the mass of the pile. The amount of additional participating soil mass, if any, is examined for each individual case.

The behavior of the proposed model is thus described by the non-linear differential equation:

$$P = \frac{S^q}{a + b.S} + C\left(\frac{\partial S}{\partial t}\right)^{0.2} + M\left(\frac{\partial^2 S}{\partial t^2}\right) \qquad (2)$$

3. Calculations

Commonly, the displacement S(t) of a pile is calculated from the forcing function P(t). Since eq. (2) is highly non-linear, this is mathematically difficult. As in the case of Statnamic testing, both P(t) and S(t) are known and there is no reason not to calculate P(t) from S(t). Because of the non-linearity, the calculation must be carried-out numerically, replacing the derivatives of S by finite differences. Because in many cases the test results are rather noisy, they are first smoothed by the moving average method.

The goal of the calculation procedure is to determine the vector of the desired parameters $\vec{\alpha} = <a,b,C,M,q>$ that will return the best fit between the calculated P(t) values and the measured ones. If $\vec{p}^M(t_i)$ is the vector of measured values and $\vec{p}(t_i)$ the vector of the calculated ones, then the vector of errors is:

$$\vec{\eta}(t_i) = \vec{p}^M(t_i) - \vec{p}(t_i) \tag{3}$$

and the error function:

$$R_n = \sum_{i=1}^{n} (\eta(t_i))^2 \tag{4}$$

Thus one has to look for the $\vec{\alpha}$ vector that will minimize the error function.

A computer program was written to perform this task automatically and uses a method similar to that called 'Gradient descent'. It starts with an initial guess for $\vec{\alpha}_0$, then changes the parameters by small steps in the direction which best reduces the error function. One gets successive converging values $\vec{\alpha}_1, \vec{\alpha}_2, \cdots$ until the change in $\vec{\alpha}_n$ is smaller then an arbitrary ε.

Once a good fit has been achieved between the calculated and measured P(t), eq. (2) will closely resemble the load-displacement curve produced during the test.

This method was found to be suitable for the task in hand, and normally did not tend to wander around local optima. In those rare cases where it did so, the program provided an option to change the parameters manually.

4. Results

The program was run on 23 files from the Statnamic database. As an illustration, three typical piles were chosen having different materials, sizes and masses. The basic details of the piles are given in Table 1. Results from these tests are presented in Figs. 1 to 3, and each figure shows the following:

1. Data measured during the test
2. The curve fitted to the above data
3. The predicted static behavior
4. The results of the static load test.

Table 1: Details of the Piles Investigated

No.	Location	Pile Material	Soil Conditions	Diameter (mm)	Length (m)	Mass (Kg)
1	McMaster	steel	clayey silt & silty sand	178	18.3	640
2	Barrie	concrete	silty sand & sandy silt	500	11.0	5,000
3	Emden	concrete	clay & sand	800	15.0	16,000

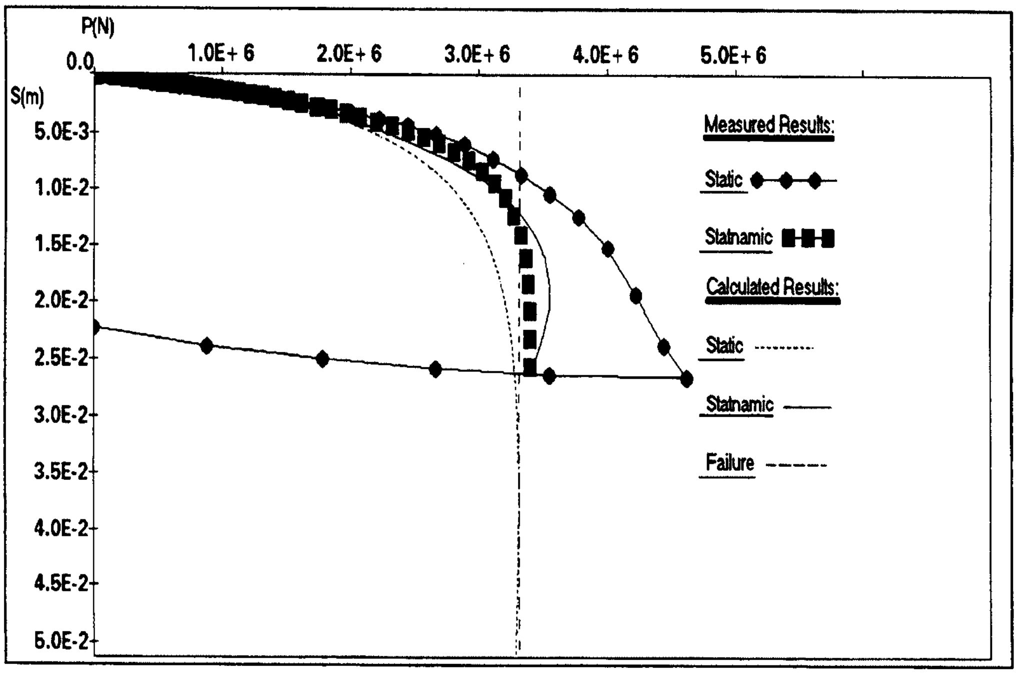

Fig. 1: McMaster (Pile No. 2)

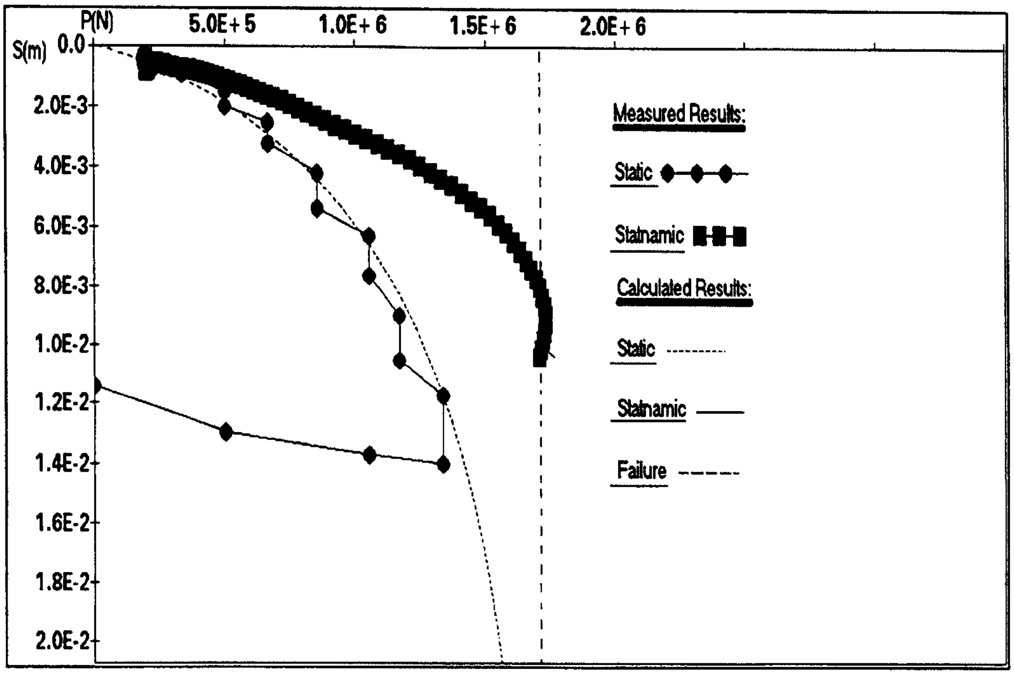

Fig. 2: Barrie (pile No. 1)

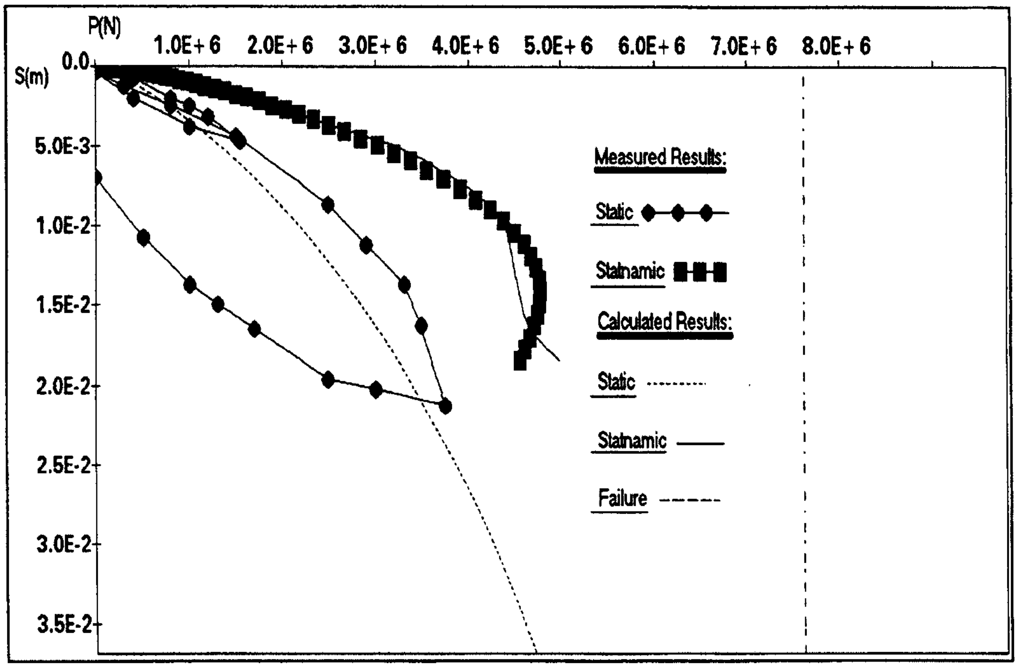

Fig. 3: Emden (pile No. 2)

5. Discussion of the Results

5.1. Applicability

As expected, it soon became apparent that the proposed mathematical model, which describes a smooth continuous curve, is unable to represent load-displacement curves that are of irregular nature. It seems that in such cases interpretation is art rather than science.

5.2. Curve-Fitting Range

While eq. (2) was found to faithfully represent the loading stage of the test, it totally failed to fit the unloading part. This may be due to the fact that eq. (1), which plots as a convex curve, cannot represent the concave trace of the unloading curve. To fully describe the hysteretic behavior typical to pile load-tests, it may take at least two equations, with more parameters than used in this work. This may increase the complexity and reduce the validity of the solution.

As a consequence of this, the curve-fitting procedure was carried out only between the origin and a point intermediate between maximum load and maximum displacement.

5.3. The q Power

As mentioned above, the value of q, that is the power of S in eq. (1), should probably lie between 0.5 and 1. Among all the cases analyzed, most of the results converged to a q value of around 0.8. This gives a curve which does simulate normal static test results, and was subsequently used as a fixed value. The number of parameters in the analysis was thus reduced to four.

5.4. The mass M

In soil dynamics, one often encounters the notion of a "participating soil mass" which moves in unison with the foundation. In all the Statnamic tests analyzed, however, this was not evident and the parameter M in all calculations converged to the bare mass of the pile. A possible explanation to this is the nature of the measured accelerations, which even after smoothing exhibited a rather "wavy" character and changed direction many times during the test.

5.5. Static Pile Behavior

Once the parameters a and b are known, the static load-displacement curve of the pile can be predicted from eq. (1). Furthermore, the predicted failure load of the pile is then given by:

$$P_{\max} = \frac{\left(\frac{4a}{b}\right)^{0.8}}{6a} \tag{5}$$

Of the three cases shown in this paper, the prediction for the Barrie case (Fig. 2) is excellent, that for Emden (Fig. 3) is good while that for McMaster is at best fair. There may be several reasons for this, but it seems that the main one is the order of testing. In Barrie, Statnamic and static tests were run on different piles, so both were subject to virgin loading. In the other two sites, the same pile was subjected to both tests. Since a pile will behave differently in reloading than in virgin loading, the discrepancy between the results is only natural.

6. Conclusions

From the results of the work presented, it appears that the proposed method offers a workable and convenient means for analyzing data from Statnamic tests. The analysis is immediate, and requires no information regarding soil conditions on site. Clearly the method needs more controlled verification (and perhaps modification) before it becomes routine.

In addition, future work in this direction should concentrate on a model that will describe the behavior of the pile during the whole test, including both downward and upward movement.

7. References

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